HW 10 CMSC 452. Morally Due May 7
THIS HW IS TWO PAGES LONG!!!!!!!!!
Throughout this HW $M_1, M_2, \ldots$ is a standard list of Turing Machines. Can also view as a list of all partial computable functions.

1. (60 points — 15 points for each part)

   (a) Let M be a Turing machine. Show that the following set is Σ_1:

   $$\{x \mid M(x)\downarrow\}$$

   (b) Describe an algorithm M such that

   $$\{x \mid M(x)\downarrow\}$$

   is undecidable.
   (HINT- Write an M such that the set

   $$\{x \mid M(x)\downarrow\}$$

   is HALT. Recall that HALT is

   $$\{e \mid M_e(e)\downarrow\}$$

)

   (c) Let M be a Turing machine. Show that the following set is Σ_1:

   $$\{y \mid \text{ there is some } x \text{ such that } M(x) = y\ \}$$

   (d) Describe an algorithm M such that

   $$\{y \mid \text{ there is some } x \text{ such that } M(x) = y\ \}$$

   is undecidable.
   (HINT- Write an M such that the set

   $$\{y \mid \text{ there is some } x \text{ such that } M(x) = y\ \}$$

   is HALT.
)

2. (40 points — 20 points each) A NATHAN program is a program that can, on each input, make 10 queries to HALT.

   (a) Is there a NATHAN program for the following problem: on input $(e_1, \ldots, e_{100})$ determine EXACTLY which e_i are such that $M_{e_i}(0) \downarrow$? (Formally the output is a bit string $(b_1, \ldots, b_{100})$ such that, for all $1 \leq i \leq 100$,

   $$M_{e_i}(0) \downarrow \text{ iff } b_i = 0.$$

)

   (b) Is there a NATHAN program for the following problem: on input n viewed as a number written in binary, output some string y such that $C(y) \geq n$ ($C(y)$ is the Kolmogorov complexity of y — the size of the smallest Turing Machine that prints out y on input 0.)